Probabilistic multiscale analysis of uncertainties in composites with disordered microstructure

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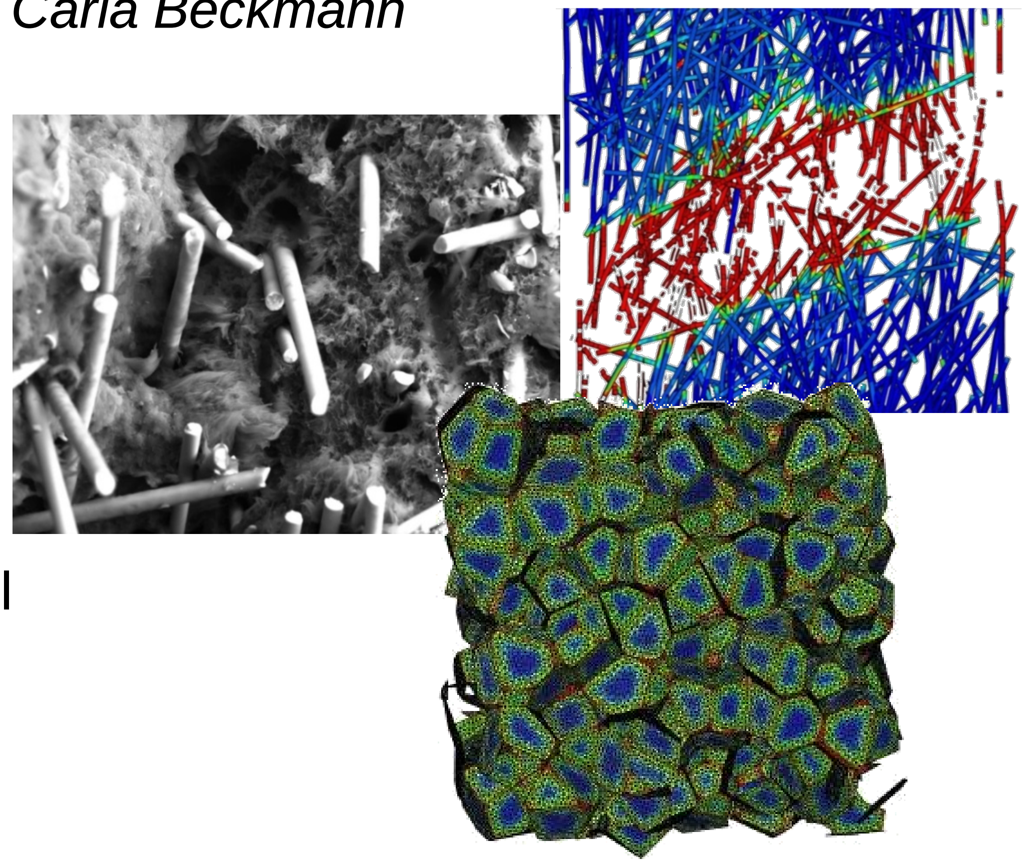

- Introduction
- Experimental observations
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- Probabilistic enhancement
- Probabilistic material model
- Case study
- Conclusions

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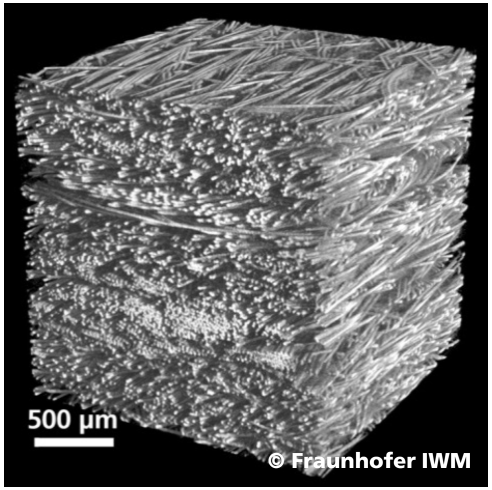
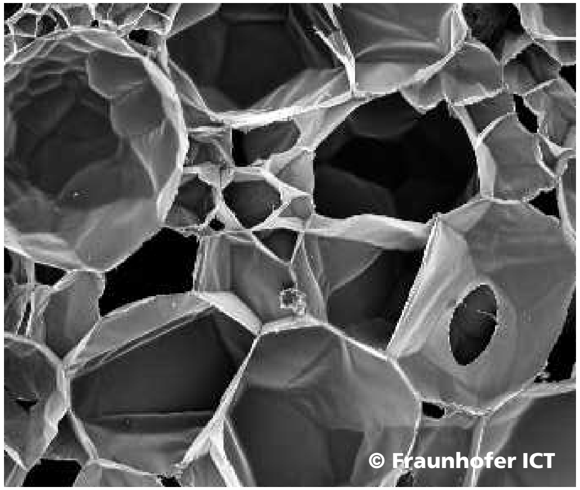
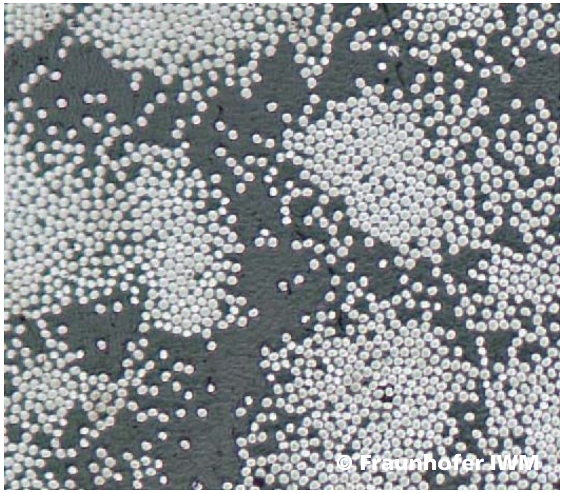
IWM

Probabilistic multiscale analysis of uncertainties
Introduction

- microstructural uncertainties in composite materials







  - uncertain fiber orientation, length, density, ...
  - uncertain particle / void size, shape, density, ...
  - uncertain constituent material properties
  - uncertain distribution of defects, delaminations, interface properties, ...

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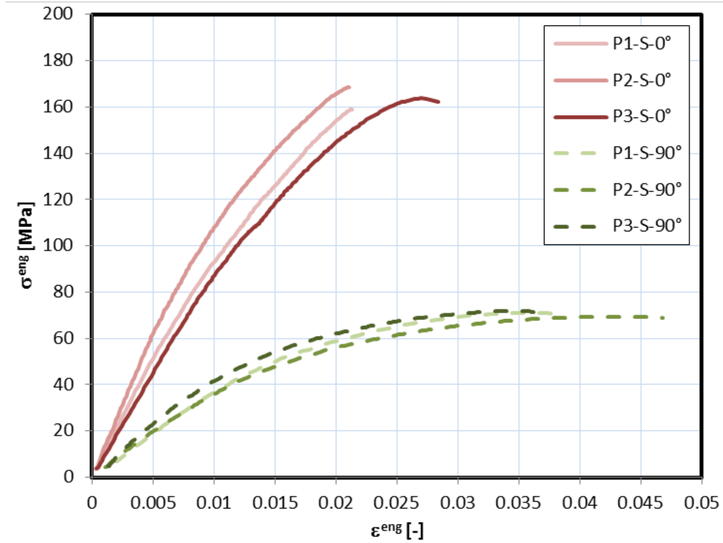
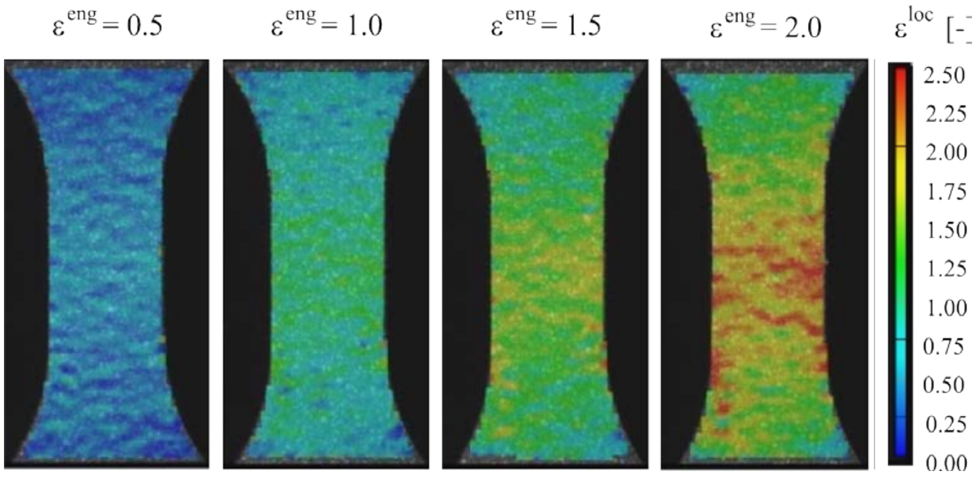
Probabilistic multiscale analysis of uncertainties
Experimental observation

- material: PA66 GF40 (compression molded)
- stress-strain response
  - overall
    - tensile tests within and perpendicular to flow direction
    - section 10 mm x 3 mm
    - overall scatter
  - local
    - grey scale correlation system (ARAMIS)
    - distinct scatter on length scales below 10 mm





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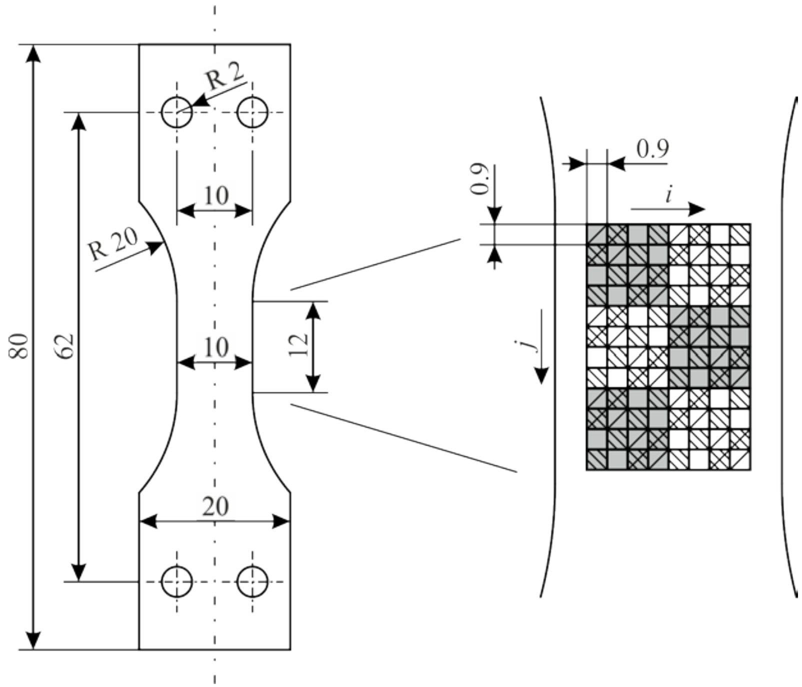
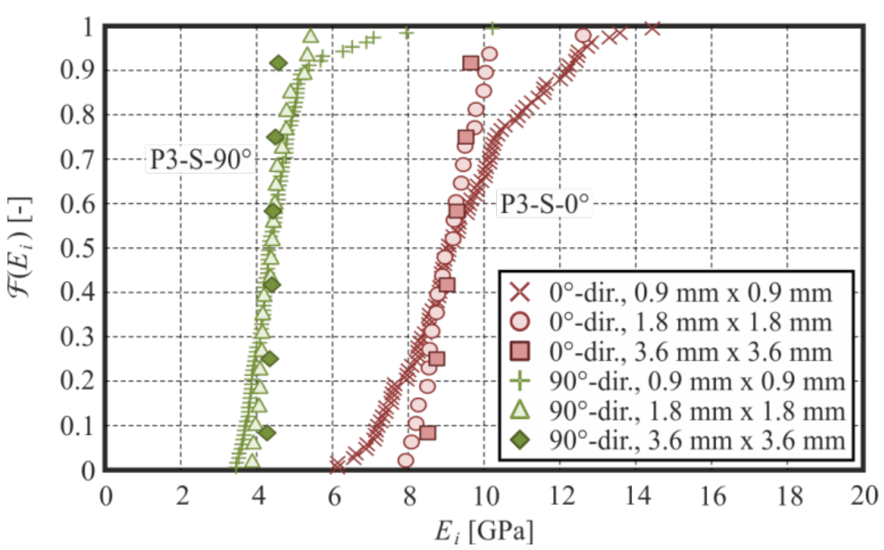
Probabilistic multiscale analysis of uncertainties

Experimental observation

- local elastic response



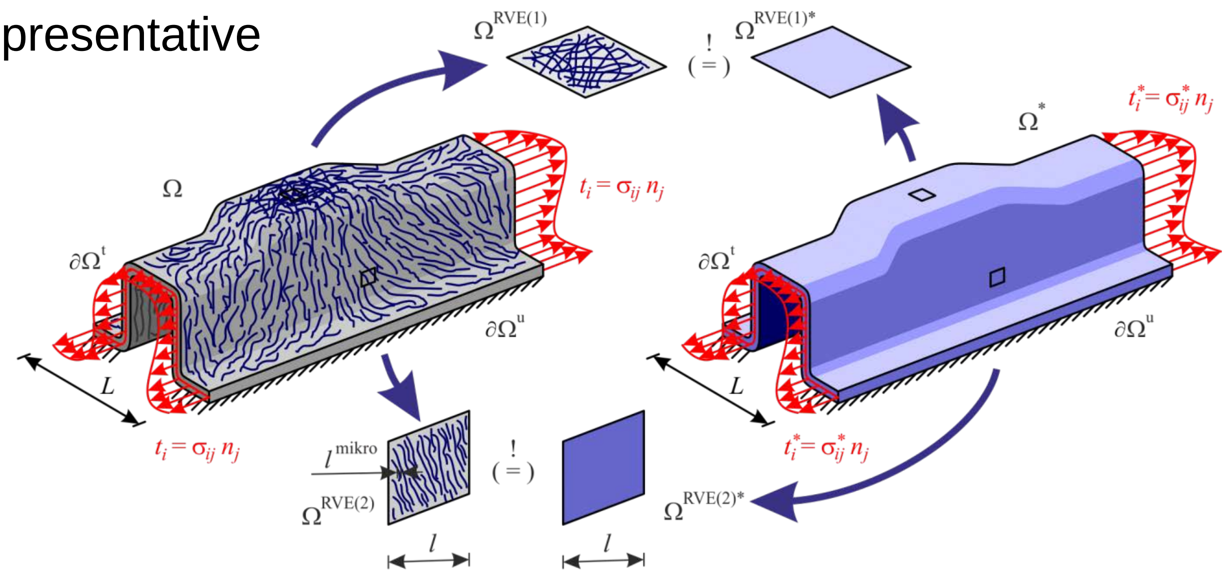


- non-negligible uncertainty in elastic constants
- dependent on size of evaluation area

Probabilistic multiscale analysis of uncertainties

Homogenization analysis

- determination of macroscopic properties by microstructural simulation
  - concept of the representative volume element (RVE)



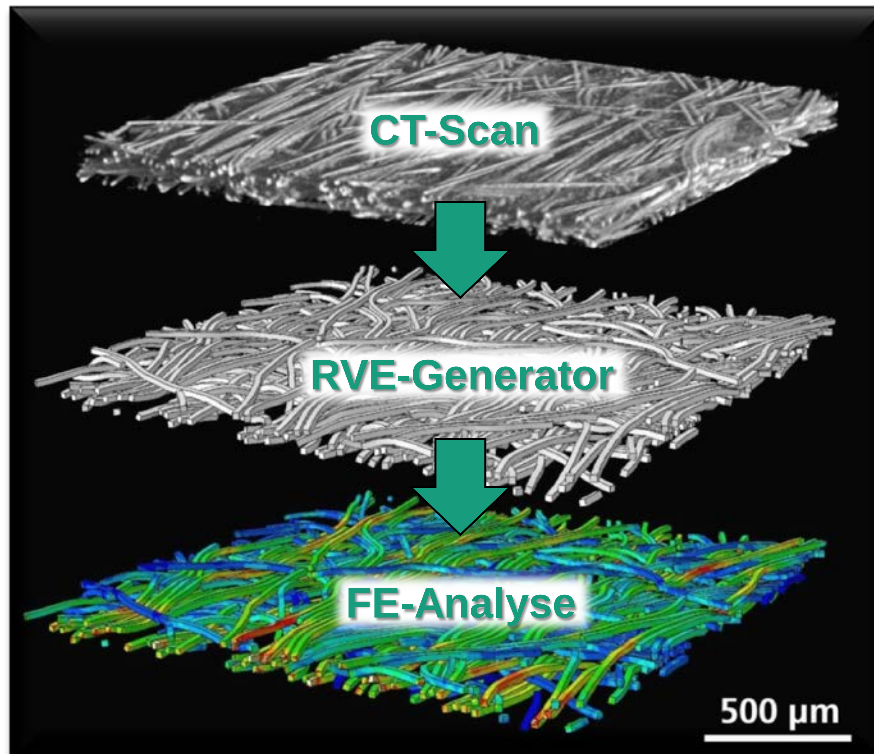
  - equivalence criteria for microscopic and macroscopic level

$$\bar{\varepsilon}_{ij} = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}}} \varepsilon_{ij} \, \mathrm{d}V = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE*}}} \varepsilon_{ij}^* \, \mathrm{d}V = \bar{\varepsilon}_{ij}^*$$

$$\bar{\sigma}_{ij} = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}}} \sigma_{ij} \, \mathrm{d}V = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE*}}} \sigma_{ij}^* \, \mathrm{d}V = \bar{\sigma}_{ij}^*$$

Probabilistic multiscale analysis of uncertainties
Homogenization analysis

- generation of representative volume elements for long fiber reinforced thermoplastic materials using CT data
  - extraction of microstructural properties from CT data
  - statistical assessment
  - RVE generation based on stochastic properties
  - assessment of equivalence of stochastic descriptors for microstructure and model
  - finite element analysis of representative volume element



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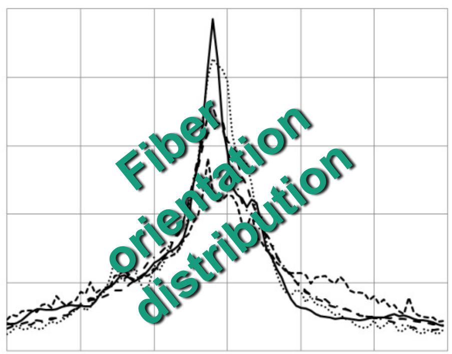
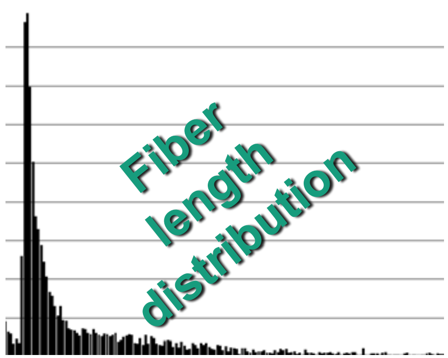
Probabilistic multiscale analysis of uncertainties
Homogenization analysis

- automatic generation of microstructure under consideration of
  - fiber length
  - fiber orientation
  - fiber volume fraction





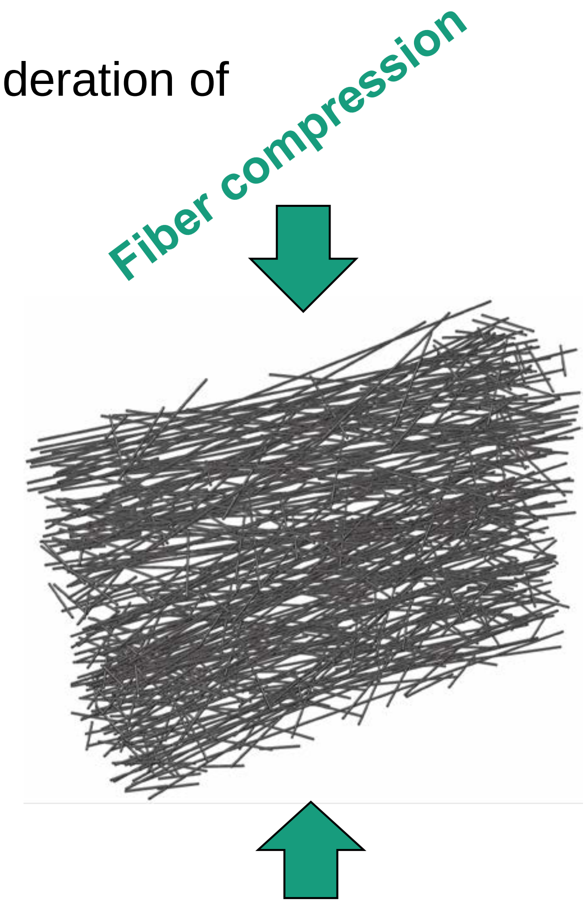
FiberGenerator

```
x11=temp_coord_x_start(m)+(1.5*element_length)
x12=temp_coord_x_end(m)+(1.5*element_length)
y11=temp_coord_y_start(m)+(1.5*element_length)
y12=temp_coord_y_end(m)+(1.5*element_length)

a1=y12-y11
a2=y22-y21
b1=x11-x12
b2=x21-x22
c1=x12*y11-x11*y12
c2=x22*y21-x21*y22

if ((a1*x21+b1*y21+c1)*(a1*x22+b1*y22+c1) .LE. 0.0) then
intersection=1
endif

if ((a2*x11+b2*y11+c2)*(a2*x12+b2*y12+c2) .LE. 0.0) then
intersection=1
endif
```



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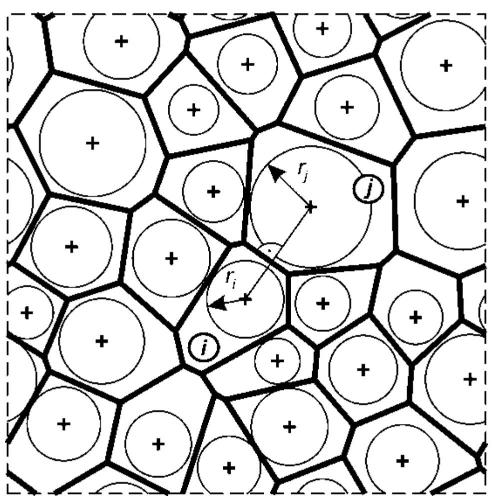
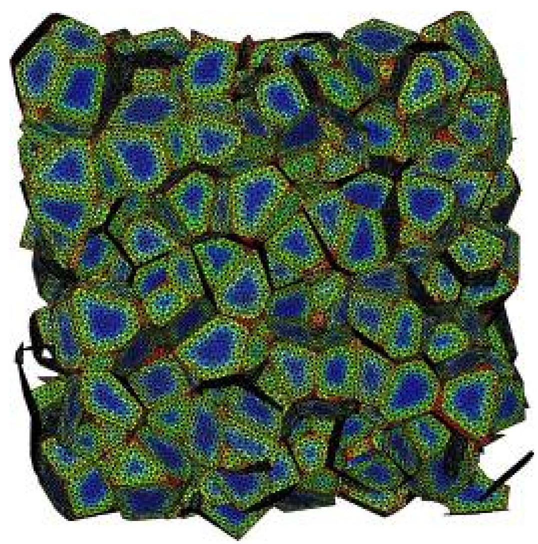
Probabilistic multiscale analysis of uncertainties
Homogenization analysis

- generation of computational foam models
  - Voronoï process in Laguerre geometry (in periodic form)

$$v_L(p_i) = \left\{ p \mid p \in R^3, r_L(p, p_i) < r_L(p, p_j), i \neq j \right\} \qquad i, j = 1, \ldots, n$$

where: $$r_L(p, p_i) = \sqrt{(x_1 - x_1^{(i)})^2 + (x_2 - x_2^{(i)})^2 + (x_3 - x_3^{(i)})^2 - r_i^2}$$





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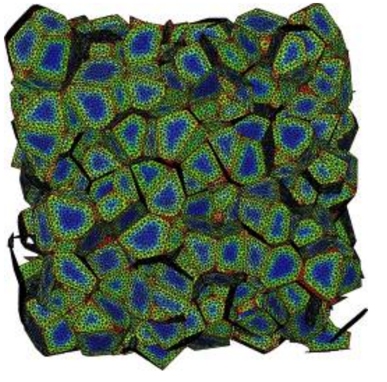
Probabilistic multiscale analysis of uncertainties
Probabilistic enhancement

- discussion
  - Hashin's requirements for the representative volume element
    - $L >> l >> l^{\mathrm{micro}}$
    - l sufficiently large to account for all possible microstructural effects
  - typical characteristic length scales e.g. solid foam core sandwiches
    - $l_{\mathrm{cell}} = 10\mu\mathrm{m}, \ldots, 2.5\mathrm{mm} \quad \Rightarrow \quad l = 0.1\mathrm{mm}, \ldots, 25\mathrm{mm}$
    - $L \geq 25\mathrm{mm}$

      → requirements might be contradictory

      → statistically representative volume element might not exist

      → probabilistic assessment required



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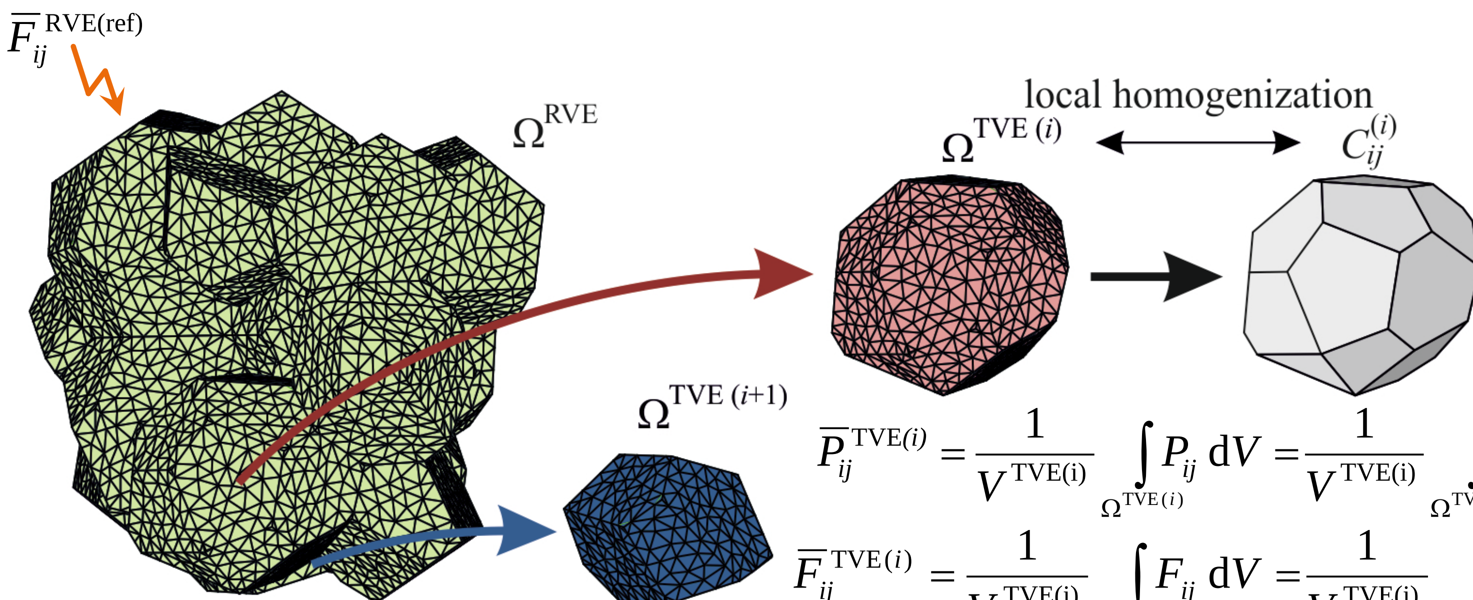
Probabilistic multiscale analysis of uncertainties
Probabilistic enhancement

- local homogenization of subsets of the representative volume element
  - subdivision of the statistically representative volume element into small scale, non-representative sub-structures
  - smallest possible sub-structures → individual cells



$$\overline{P}_{ij}^{\mathrm{TVE}(i)} = \frac{1}{V^{\mathrm{TVE(i)}}} \int_{\Omega^{\mathrm{TVE}(i)}} P_{ij}\, \mathrm{d}V = \frac{1}{V^{\mathrm{TVE(i)}}} \int_{\Omega^{\mathrm{TVE}(i)*}} P_{ij}^{*}\, \mathrm{d}V^{*}$$

$$\overline{F}_{ij}^{\mathrm{TVE}(i)} = \frac{1}{V^{\mathrm{TVE(i)}}} \int_{\Omega^{\mathrm{TVE}(i)}} F_{ij}\, \mathrm{d}V = \frac{1}{V^{\mathrm{TVE(i)}}} \int_{\Omega^{\mathrm{TVE}(i)*}} F_{ij}^{*}\, \mathrm{d}V^{*}$$

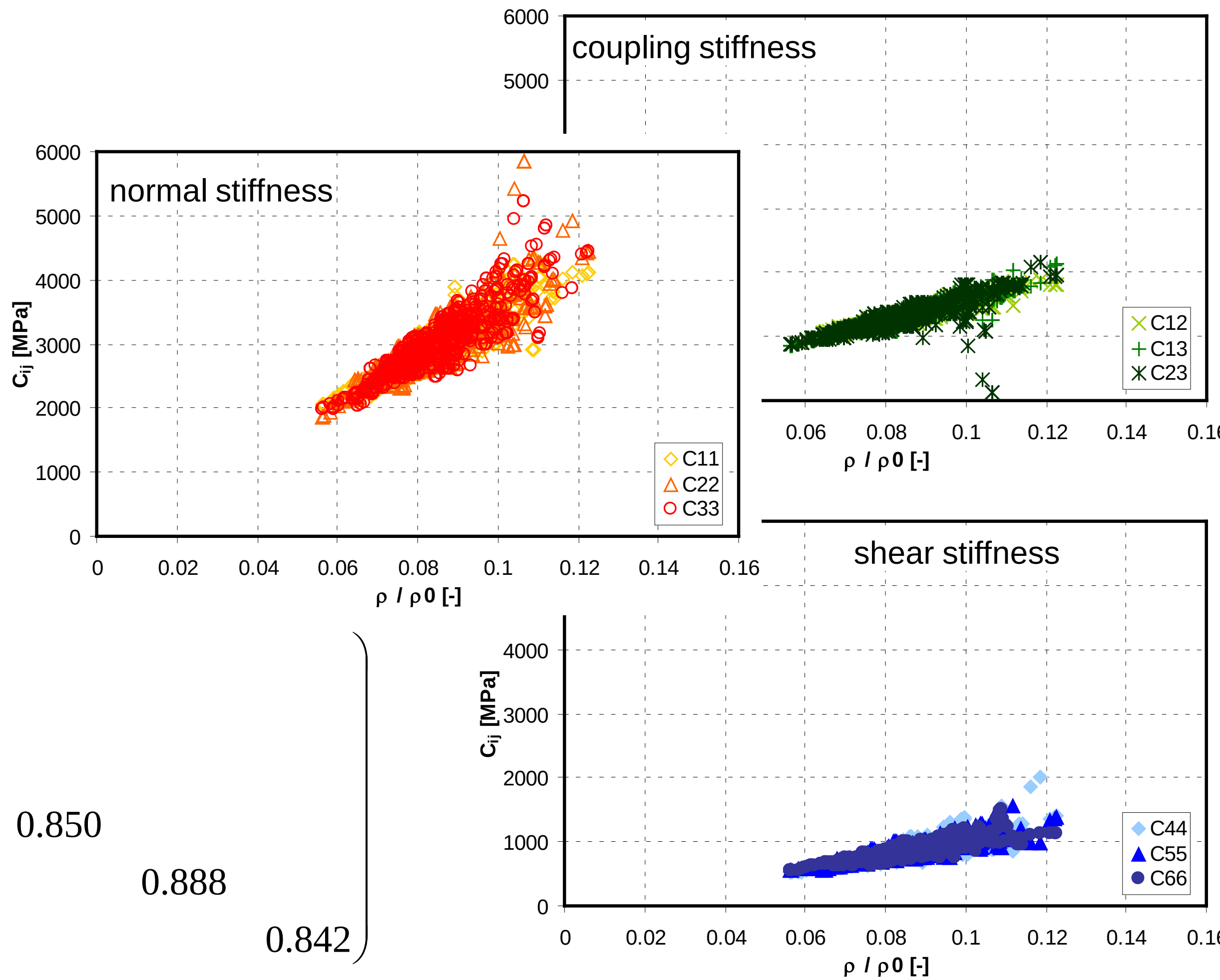
Probabilistic multiscale analysis of uncertainties
Probabilistic enhancement

- numerical results
  - raw data base

→ isotropic effective response

→ results governed by local density

$$R\left(C_{ij}^{\mathrm{cell}}, \frac{\rho^{\mathrm{cell}}}{\rho_0}\right) = \begin{pmatrix} 0.896 & 0.881 & 0.881 & & & \\ & 0.895 & 0.860 & & & \\ & & 0.913 & & & \\ & & & 0.850 & & \\ & & & & 0.888 & \\ \mathrm{sym.} & & & & & 0.842 \end{pmatrix}$$



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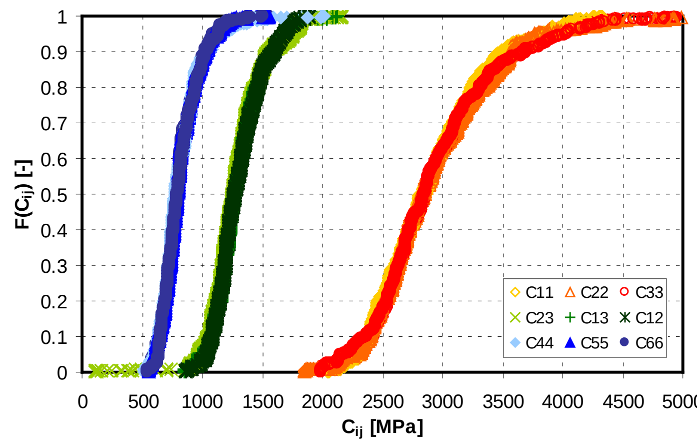
Probabilistic multiscale analysis of uncertainties

Probabilistic enhancement

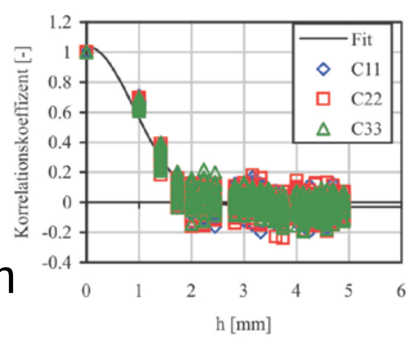
- probability distributions
  - stochastic evaluation
    - → asymmetric distributions
    - → assessment in terms of stochastic parameters may be insufficient

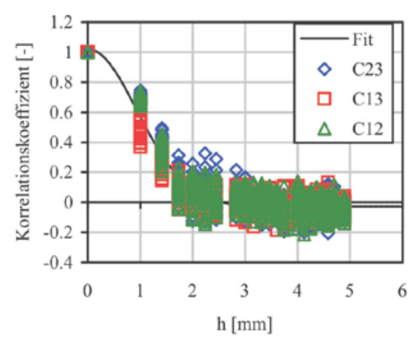


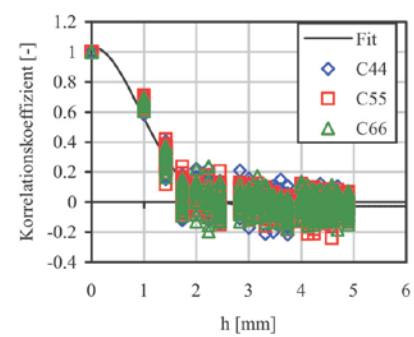
- autocorrelation

$$R(\Delta C_{ij},h)=\frac{E(\Delta C_{ij}h)-E(\Delta C_{ij})\,E(h)}{\left(V(\Delta C_{ij})\,V(h)\right)^{1/2}}$$

→ spatial correlation vanishes for h > 2 mm







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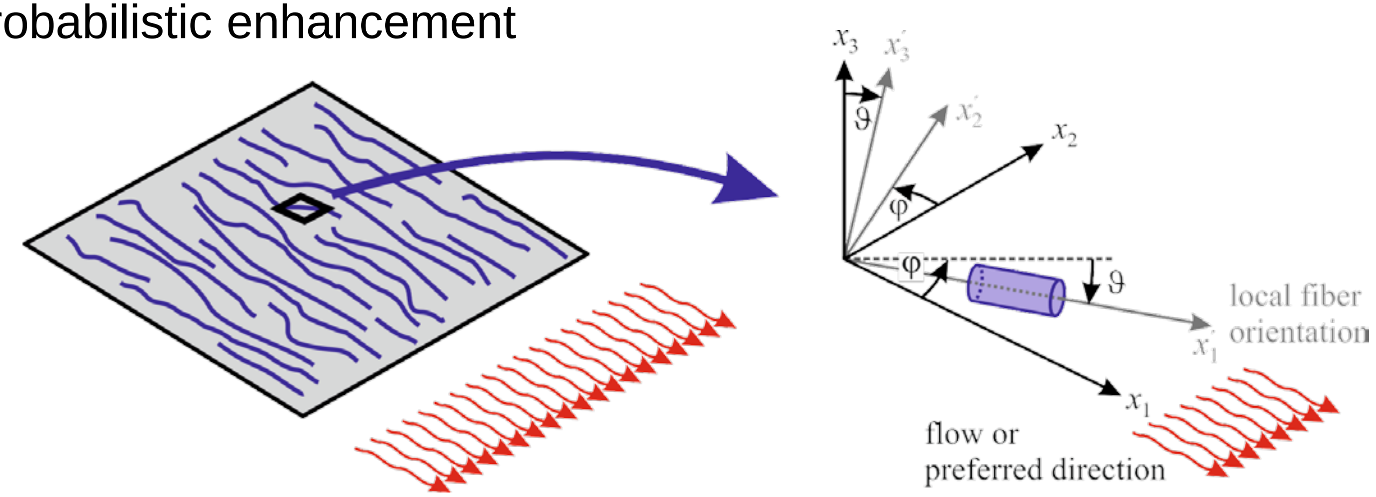
Probabilistic multiscale analysis of uncertainties
Probabilistic material model

- objective: probabilistic elasticity model accounting for local microstructural uncertainties
- three step procedure

  (1) basic single-fiber model

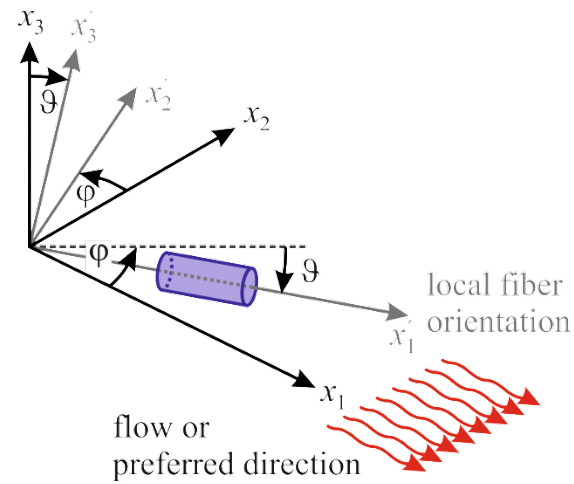
  (2) generalization to multi-fiber model

  (3) probabilistic enhancement



Probabilistic multiscale analysis of uncertainties
Probabilistic material model

- single-fiber model: approximation by rules of mixture



$$\begin{pmatrix} \varepsilon'_{11} \\ \varepsilon'_{22} \\ \varepsilon'_{33} \\ 2\varepsilon'_{23} \\ 2\varepsilon'_{13} \\ 2\varepsilon'_{12} \end{pmatrix} = \begin{pmatrix} 1/E'_1 & -\nu'_{21}/E'_2 & -\nu'_{31}/E'_3 & 0 & 0 & 0 \\ -\nu'_{12}/E'_1 & 1/E'_2 & -\nu'_{32}/E'_3 & 0 & 0 & 0 \\ -\nu'_{13}/E'_1 & -\nu'_{23}/E'_2 & 1/E'_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G'_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G'_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G'_{12} \end{pmatrix} \begin{pmatrix} \sigma'_{11} \\ \sigma'_{22} \\ \sigma'_{33} \\ \sigma'_{23} \\ \sigma'_{13} \\ \sigma'_{12} \end{pmatrix}$$

where:

$$E'_1 = \rho_f E_f + (1-\rho_f)E_m \quad , \quad E'_2 = \frac{E_f E_m}{\rho_f E_m + (1-\rho_f)E_f} \quad , \quad E'_3 = \frac{E_f E_m}{\rho_f E_m + (1-\rho_f)E_f}$$

$$G'_{23} = \frac{E_2}{2(1+\nu_{23})} \quad , \quad G'_{13} = \frac{G_f G_m}{\rho_f G_m + (1-\rho_f)G_f} \quad , \quad G'_{12} = \frac{G_f G_m}{\rho_f G_m + (1-\rho_f)G_f}$$

$$\nu'_{23} = \rho_f \nu_f + (1-\rho_f)\nu_m \quad , \quad \nu_{13} = \rho_f \nu_f + (1-\rho_f)\nu_m \quad , \quad \nu'_{12} = \rho_f \nu_f + (1-\rho_f)\nu_m$$

$$\nu_{32} = \nu_{23}\frac{E_3}{E_2} \quad , \quad \nu_{31} = \nu_{13}\frac{E_3}{E_1} \quad , \quad \nu_{21} = \nu_{12}\frac{E_2}{E_1}$$

Probabilistic multiscale analysis of uncertainties
Probabilistic material model

- generalization to multi-fiber model
  - transformation of the single fiber solution into the global system

$$C^{\mathrm{s}}_{ijkl} = a_{ip}\, a_{jq}\, a_{kr}\, a_{ls}\, C^{\mathrm{s}'}_{pqrs} \qquad \text{with:} \qquad a_{ij} = \begin{pmatrix} \cos\varphi\cos\vartheta & -\sin\varphi & \cos\varphi\sin\vartheta \\ \sin\varphi\sin\vartheta & \cos\varphi & \sin\varphi\sin\vartheta \\ -\sin\vartheta & 0 & \cos\varphi \end{pmatrix}$$

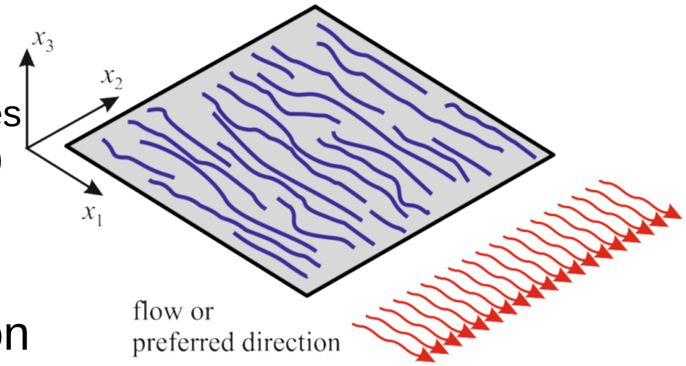
  - individual fiber angles subject to variations

$$f_\varphi(\varphi) = \frac{1}{c_\varphi}\frac{1}{s_\varphi (2\pi)^{1/2}} e^{-\frac{1}{2}\frac{\varphi^2}{s_\varphi^2}}$$

$$f_\vartheta(\vartheta) = \frac{1}{c_\vartheta}\frac{\cos\vartheta}{s_\vartheta (2\pi)^{1/2}} e^{-\frac{1}{2}\frac{\vartheta^2}{s_\vartheta^2}}$$

s_φ, s_ϑ: orientation variabilities (material parameters)

c_φ, c_ϑ: correction factors



  - ensemble average of the single fiber solution

$$C^{\mathrm{m}}_{ijkl} = \mathrm{EV}(C^{\mathrm{s}}_{ijkl}) = \int_{\varphi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\vartheta=-\frac{\pi}{2}}^{\frac{\pi}{2}} C^{\mathrm{s}}_{ijkl}(\varphi,\vartheta)\; f_\varphi(\varphi)\; f_\vartheta(\vartheta) \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

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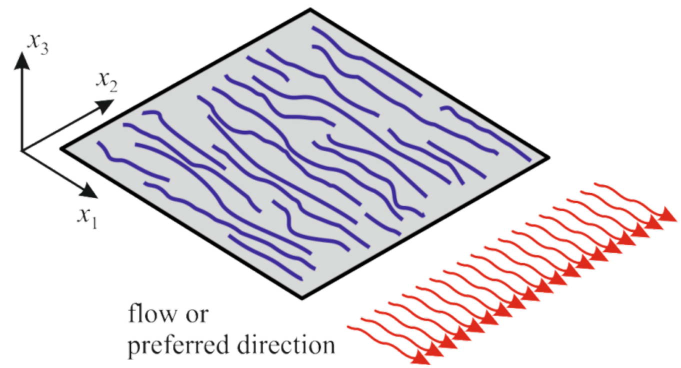
Probabilistic multiscale analysis of uncertainties
Probabilistic material model

- probabilistic enhancement
  - ensemble average of the single fiber solution

$$C^{\mathrm{m}}_{ijkl}(\rho_{\mathrm{f}}, s_\varphi, s_\vartheta) = \mathrm{EV}(C^{\mathrm{s}}_{ijkl}) = \int\limits_{\varphi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int\limits_{\vartheta=-\frac{\pi}{2}}^{\frac{\pi}{2}} C^{\mathrm{s}}_{ijkl}(\rho_{\mathrm{f}}, \varphi, \vartheta)\; f_\varphi(s_\varphi, \varphi)\; f_\vartheta(s_\vartheta, \vartheta) \sin\vartheta \,\mathrm{d}\varphi \,\mathrm{d}\vartheta$$

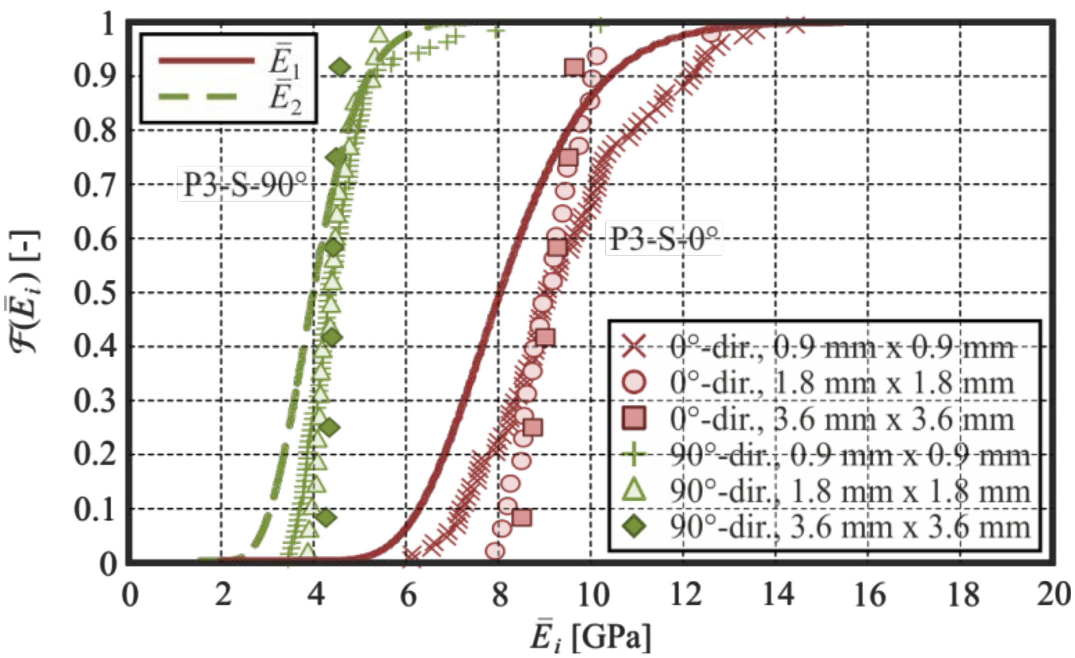
    - still deterministic model
    - providing a deterministic stiffness for prescribed standard deviations of the fiber angles



  - in real components
    - local microstructure uncertain
    - here: local fiber density and local fiber angles uncertain
    - ρ_f, s_φ, s_ϑ: stochastic (random) variables,
      supplied with probability distribution

Probabilistic multiscale analysis of uncertainties
Probabilistic material model

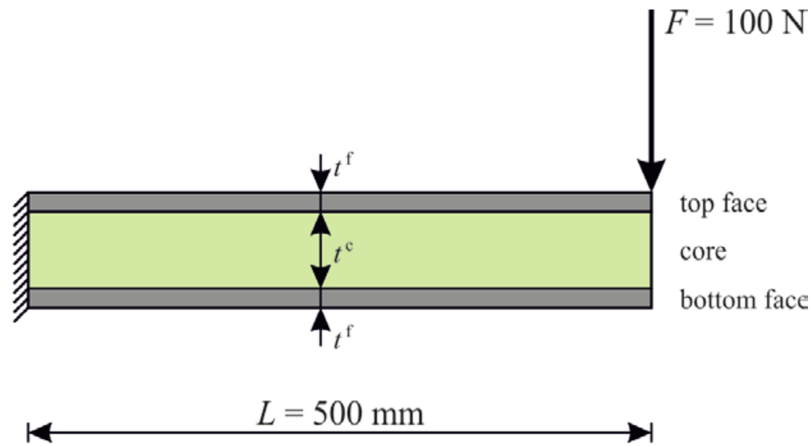
- comparison of experimental observation and numerical prediction



→ good agreement of experimental and numerical results
→ differences in E_1 might be caused by variation in ρ_f

Probabilistic multiscale analysis of uncertainties
Case study

- single edge clamped sandwich beam
  - aluminum face sheets
    - $t^{\mathrm{f}} = 1$ mm
    - $E^{\mathrm{f}} = 70$ GPa, $\nu^{\mathrm{f}} = 0.3$
  - aluminum foam core
    - $t^{\mathrm{f}} = 25$ mm
  - core material:
    random field according to
    homogenization results
    ($r^{\mathrm{TVE}} = 4$ mm, 256 cells)
- linear elastic finite element analysis



$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ & & C_{33} & C_{34} & C_{35} & C_{36}\\ & & & C_{44} & C_{45} & C_{46}\\ & & & & C_{55} & C_{56}\\ \text{sym.} & & & & & C_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{pmatrix}$$

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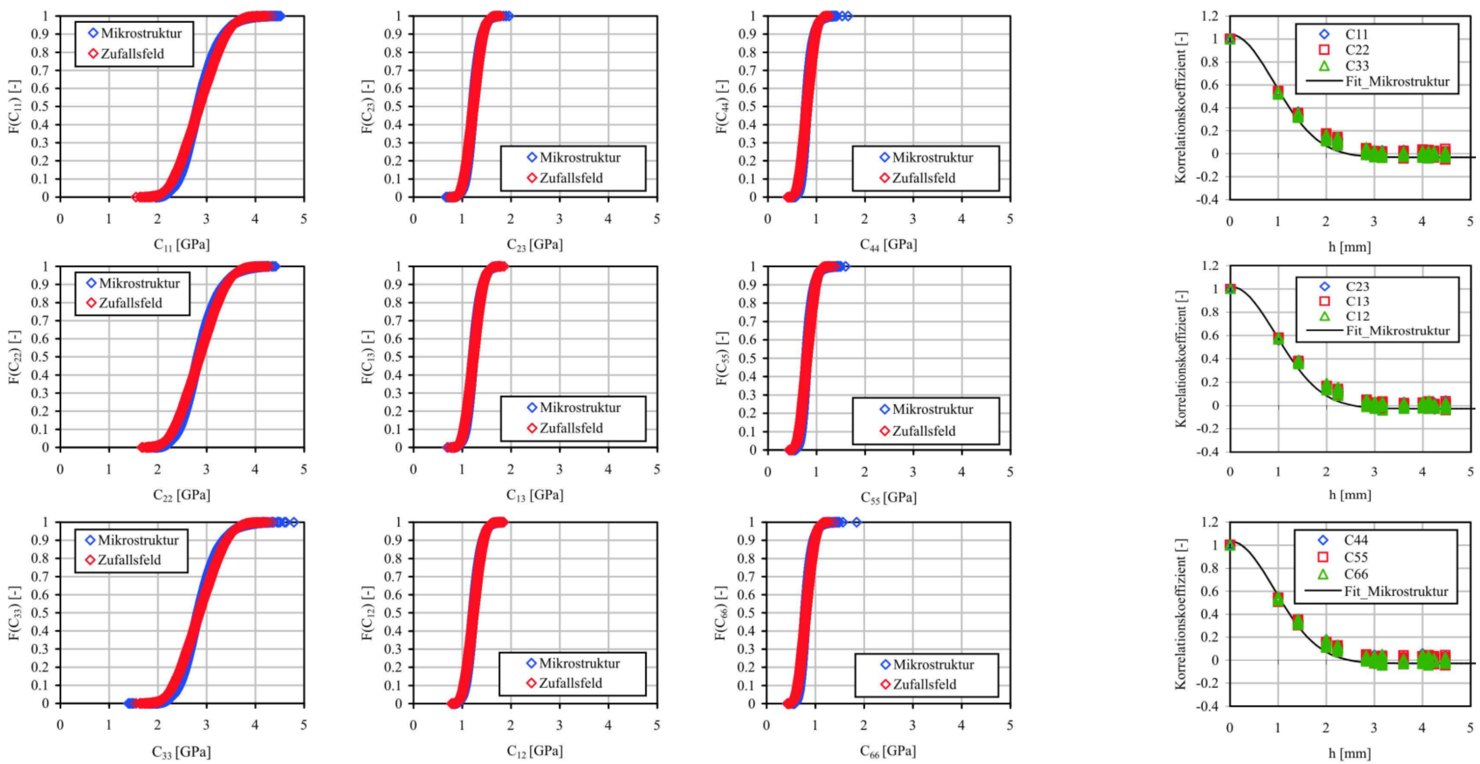
Probabilistic multiscale analysis of uncertainties

Case study

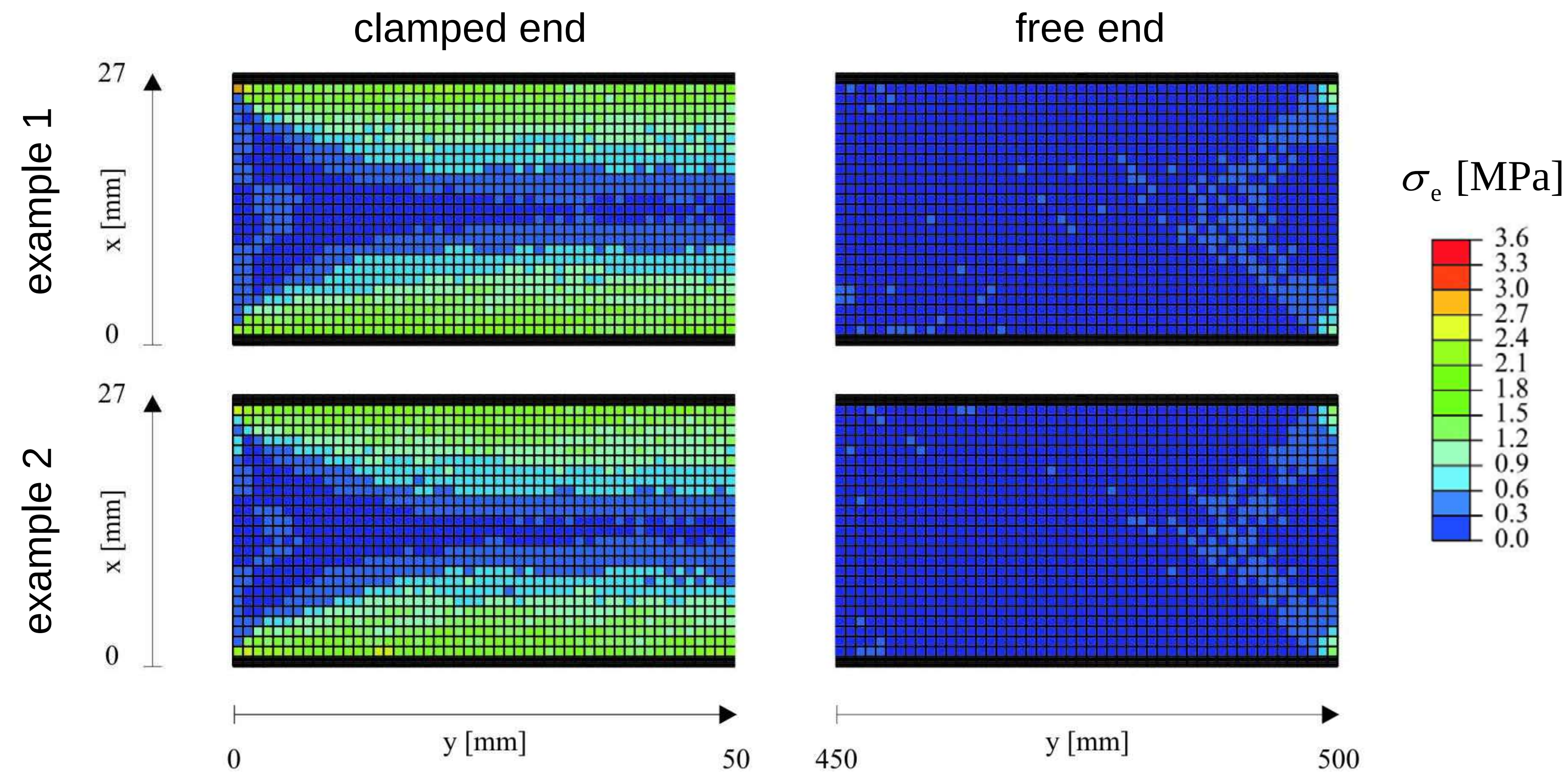
- validation: uncertainty in homogenization results and random field



→ homogenization results and random field approximation in good agreement

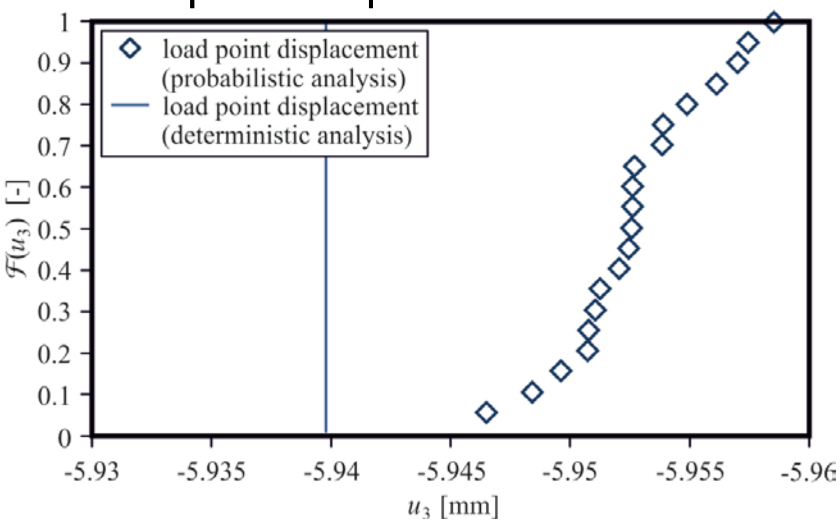
Probabilistic multiscale analysis of uncertainties
Case study

- stress results

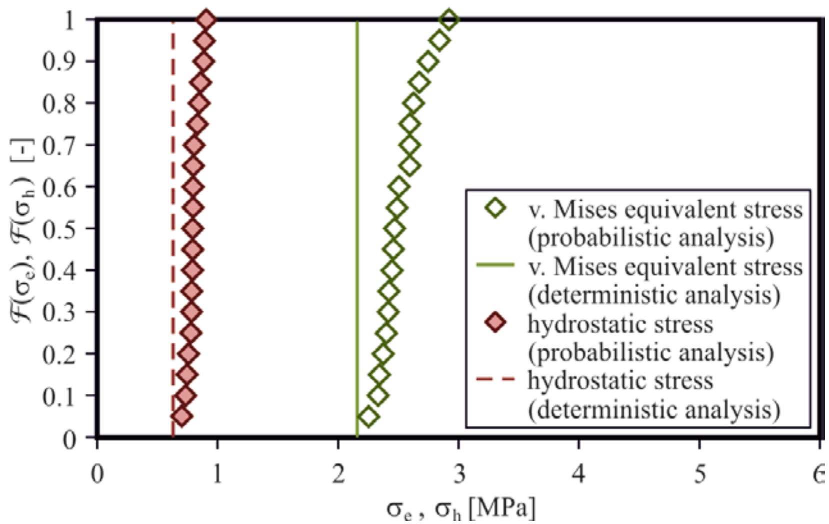


Probabilistic multiscale analysis of uncertainties
Case study

- probability distributions
  - stiffness:
    load point displacement



  - strength:
    maximum core stresses



→ only limited uncertainty
→ overall deformation property governed by average properties

→ more distinct variability
→ underestimation of possible load maximum by deterministic analysis

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Probabilistic multiscale analysis of uncertainties
Conclusions

- microstructural uncertainties in composite materials
  - may lead to distinct material uncertainties on the macroscopic level
  - arise if the characteristic microstructural length scale is not vanishingly small compared to the (smallest) macroscopic characteristic length
- numerical scheme for prediction of structural uncertainties due to material uncertainties caused by microstructural geometric uncertainties
  - probabilistic homogenization procedure
  - probabilistic finite element model
  - probabilistic structural analysis based on random field model
  - especially strength limits affected

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Probabilistic multiscale analysis of uncertainties
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